Calculation model of uniform media filtration capacity

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Abstract

Filtration is an efficient method for turbidity removal and widely used for water and gas purification. The process of filtration is complicated, and although the basic principles concerning filtration were brought forward decades ago, there is a gap between the theories and their application. Filtration is still an active field for theoretical and experimental research, and there is plenty of room for improvement in this area. In this paper, through the analysis of the uniform media filtration, the capillary model is used to determine the flow pattern in the packed bed, and then the fluid shear force distribution in the capillary is formulated. The physical-chemical forces including van der Waals attraction force, electric double-layer force, Born repulsion force and structural force between the particle and the capillary wall are calculated by analytical equations. By equilibrium analysis of the forces acting on a particle adhered on the capillary wall and appropriate simplification, the analytical equation of the porosity in the saturated filter bed is obtained, from which the maximum entrapment capacity during the filtration process can be achieved. Moreover, the filtration capacity under certain operations are also presented and discussed. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Filtration; Entrapment capacity; Capillary model; Porosity; Hydraulic slope

1. Introduction

Filtration, which has been widely used in water and gas purification, is an efficient method for the removal of suspended particles. It is the last process in potable water treatment, which will influence the tap water quality directly. In the filtration process, the suspended particles are entrapped by the filter bed and clean fluid can be achieved.

The process of filtration is very complicated, and although the basic principles concerning filtration were brought forward decades ago [1–4], there is a gap between the theories and their application. The capture mechanisms and surface interaction between suspended particles and filter grain have been thoroughly studied by many researchers [5–12]. However, the filtration theories are always concern about complex numerical solutions, especially for the trajectory analyses [4–6], and deviation will occur when these theories are used to predict the deposition rate and filtration capacity [3,4,13,14]. Since filtration is an easy

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and inexpensive way to remove the suspended particles, it is a promising process worth further study. Nowadays, filtration is still an active field for theoretical and experimental research, and there is plenty of room for improvement in this area.

Based on the porous media filtration, Jing et al. proposed a capillary model, which was then used to calculate the turbidity removal and head loss process in uniform media filtration [15,16]. In this paper, the capillary model is combined with the surface forces to analyze the saturation stage of uniform media packed bed filtration. Firstly, the fluid state and shear force distribution in the capillary is studied. Then, the physicochemical forces including London–van der Waals attraction force, electric double-layer force, Born repulsion force and structural (or hydration) force between the particle and the capillary wall are calculated by analytical equations. Thirdly, the analytical equation of the saturation porosity in the filter bed is obtained through appropriate simplification and equilibrium analyses of the interaction forces of a particle adhered to the capillary wall. The maximum entrapment capacity of the filter bed can be achieved from the equation formulated, i.e. the difference of porosity plus the overall volume of the filter bed. In the last section of this paper, some results under certain operation conditions are presented and discussed.

2. Capillary model

2.1. Establishment of the capillary model

Water containing suspended particles transfers through the porous media as laminar flow in the packed granular bed filtration processes, and the particles interact with the filter grain while moving with the water. When the interaction force is strong enough to bond the particle to the grain surface, the particle is removed from water. It can be seen from Fig. 1 that the fluid shear force in the pore of filter bed is not uniform, thus the particle will deposit on the lower shear force position first (see Fig. 1, position a). In Fig. 1 the number 3, 2 and 1 circles are the maximum

inscribed circle, the equivalent area circle and the minimum circumscribed circle of the pore among the filter grains, respectively, while position a and b are the place of the minimum and maximum fluid shear force, respectively. (The calculation equation of fluid shear force is shown as Eq. (8)).

The forces on the surface of each pore will be about equal when the filter bed approaching its saturation, then the pores in the bed can be considered as circular shaped capillaries. The following assumptions and equations can be formulated through comparison of capillary hypothesis with actual filtration system [5]:

$$\varepsilon = n \times \frac{\pi}{4} d_c^2 \quad (1)$$

$$f = n \times \pi d_c \quad (2)$$

where ε is the porosity of the filter bed, f is the specific surface area of the filter bed, m^{-1} , n is the number of capillaries per unit surface area of the filter bed, m^{-2} , d_c is the capillary diameter, m . For non-spherical filter grain, the specific surface area can be expressed as [15]:

$$f = \frac{6\alpha(1 - \varepsilon)}{d_e} \quad (3)$$

where α is the surface shape factor and d_e is the equivalent diameter of a filter grain, m .

Substituting Eqs. (1) and (2) into Eq. (3), the following equation can be obtained:

Fig. 1. The minimum cross-section of flow in uniform spherical filter bed (the hatched area).

$$d_c = \frac{2\varepsilon}{3\alpha(1-\varepsilon)} d_e \quad (4)$$

$$n = \frac{9\alpha^2(1-\varepsilon)^2}{\pi \varepsilon d_e^2} \quad (5)$$

The average flow rate in each capillary must be equal to that of the pores in the porous filter media, that is:

$$\bar{u} = \frac{Q}{A_e} \quad (6)$$

where $\bar{u}$ is the average flow rate in capillaries, ms^{-1} , Q is the apparent filtration velocity, ms^{-1} , A_e is the pore area per unit area of filter medium:

$$A_e = \pi \left(\frac{d_c}{2} \right)^2 n = \varepsilon$$

2.2. Shear force distribution in pressurized circular shaped capillary

As mentioned in Section 2.1, the flow in the capillary is considered as laminar flow. For the laminar flow in a circular shaped capillary whose radius is r_0 , the velocity distribution can be expressed as:

$$u = \frac{\gamma J}{4\mu} (r_0^2 - r^2),$$

and the average velocity is [17]:

$$\bar{u} = \frac{\gamma J}{32\mu} d^2 \quad (7)$$

where d is the diameter of the capillary, m , r is the distance from an arbitrary point to the center of the capillary, m , γ is the fluid density, kN m^{-3} , and μ is the dynamic viscosity of the fluid (at temperature of 20 °C μ is $1.002 \times 10^{-3} \text{ N s m}^{-2}$ for water). J is the hydraulic slope: $J = h_f/l$, where h_f is the hydraulic loss, m , l is the capillary length, m .

For the uniform flow in the pressurized circular shaped capillary, the relation between the shear force τ and the hydraulic loss can be given as:

$$\tau = \gamma \times \frac{r}{2} \times J,$$

and the average hydraulic loss of the laminar flow in the capillary can be written as:

Fig. 2. The shear force distribution of uniform flow in pressurized circular shaped capillary.

$$\bar{h}_f = J \times l$$

Thus the shear force distribution in the circular shaped capillary can be obtained from Eq. (7):

$$\tau = \gamma \times \frac{r}{2} \times J = \gamma \times \frac{r}{2} \times \frac{32\mu\bar{u}}{\gamma d^2} = \frac{16 \times \mu \times \bar{u} \times r}{d^2} \quad (8)$$

It can be concluded from Eq. (8) that the shear force in the center of the capillary equals zero, and it is in direct proportion to the distance between the capillary centre and the calculation position. The maximum value $8\mu\bar{u}/d$ is reached on the inner wall of capillary, and the shear force distribution in the capillary is shown in Fig. 2.

Supposing that the distance between the capillary wall and the particle is z , then $r = d_c/2 - z$, and the fluid shear force in position z can be calculated by the following equation:

$$\begin{aligned} \tau &= \frac{16 \times \mu \times \bar{u} \times \left(\frac{d_c}{2} - z \right)}{d_c^2} \\ &= 8 \times 10^{-3} \times \bar{u} \times (d_c^{-1} - 2zd_c^{-2}) \end{aligned} \quad (9)$$

where, μ is taken as $1.002 \times 10^{-3} \text{ N s m}^{-2}$.

3. Static interaction forces between the particle and capillary wall

During the filtration process, various physical and chemical forces affect on the attachment of particles onto the wall of the filter bed pores. These forces can be divided into two groups according to their acting region [7]. The London–van der Waals attraction force (F_L) and the electric double-layer force (F_e) (either attractive or

repulsive) are called long-range forces, because their influences exist even when the distance between the particle and the capillary wall is about 100 nm. The Born repulsion force (F_B) and structural (or hydration) force (F_h) are termed as short-range forces due to their influence on particles being dominant only if the particles are less than 5 nm away from the interaction surface. During the saturation course of the filter bed, these forces will affect the adsorption and desorption of the particles on the pore wall. The equations governing these forces are given below: Retarded London–van der Waals force [9,13]:

$$F_L = \frac{\left(\frac{A_{\text{cwp}}}{6}\right) \left[1 + 28\left(\frac{z}{\lambda_e}\right)\right]}{\left\{z^2 \left[1 + 14\left(\frac{z}{\lambda_e}\right)\right]^2\right\}} \quad (10)$$

Electrical Double-layer force [10,13]:

$$F_e = -64\pi a_p \varepsilon \kappa \left[\frac{k_B T}{Ze}\right]^2 \tanh\left[\frac{Ze\psi_1}{4k_B T}\right] \tanh\left[\frac{Ze\psi_2}{4k_B T}\right] \times \exp(-\kappa z) \quad (11)$$

Born force [11,13]:

$$F_B = \frac{-A_{\text{cwp}} a_p \sigma_i^6}{(180z^8)} \quad (12)$$

Hydration force [12,13,18]:

$$F_h = -2\pi a_p K h \exp\left(\frac{-z}{h}\right) \quad (13)$$

where F_B , F_e , F_h , F_L are the Born force, electrical double-layer force, hydration force and London–van der Waals force, respectively, (Nm s^{-2}), a_c , a_p are the radii of filter grains and particles, respectively, (m), A_{cwp} is the Hamaker constant when a particle (p) and a filter grain (c) are separated by water (w) ($\text{Nm}^2 \text{s}^{-2}$), z is the distance between the surfaces of a particle and a filter grain (m), λ_e is the characteristic wavelength of the interaction (m), defined as $2\pi c/\omega_v$, c is the velocity of light, $2.9979 \times 10^8 \text{ m s}^{-1}$, ω_v is the dispersion frequency, for most materials λ_e takes a value of around 100 nm, κ is the inverse Debye length (m^{-1}), which can be approximated as [19]:

$$\kappa^{-1} \approx \frac{2.8}{\sqrt{I}} \text{ nm},$$

I is the ionic strength, mole l^{-1} . k_B is the Boltzmann's constant, $1.3807 \times 10^{-23} \text{ J K}^{-1}$, T is the temperature of the suspension, K, Z is the charge number of the electrolyte used, e is the charge of an electron, $1.6022 \times 10^{-19} \text{ C}$, ψ_1 and ψ_2 are the surface potential of particles and filter grains, respectively, σ_i is the collision diameter (m), K_1 and h are empirical constants, ε is the permittivity of the suspension ($\varepsilon = \varepsilon_r \times \varepsilon_0$), ε_0 is the permittivity of the vacuum and ε_r is the dielectric constant of the suspension.

The above formulae have been derived based on assumptions that the surface of capillary wall in the filter bed is as a flat surface when compared to a particle. The retarded London–van der Waals force given in Eq. (10) is reliable even when the separations are about 20% of the particle radius and the electrical double-layer force given in Eq. (11) is valid only for the following conditions: (i) the absolute surface potentials are less than about 60 mV; (ii) the electrolyte in the suspension is univalent. For larger surface potentials, the elliptic integral formulae should be used [13].

The calculation of the Hamaker constant during the filtration of water is described as follows: The value of Hamaker constant, which is about 10^{-19} – 10^{-20} J for most cases, depends on the density and the polarizability of the material. For two mass point of the same material the Hamaker constant can be expressed as $A_{131} = (A_{11}^{1/2} - A_{33}^{1/2})^2$, where A_{131} is the effective Hamaker constant of the particle in the medium, and A_{11} and A_{33} are the Hamaker constant of the mass point and the medium, respectively, [19,20]. The filter grain is quartz sand for most cases of potable water filtration, and assuming the suspended particles are also composed of silica, then the Hamaker constant between the particle and the capillary wall can be approximated by the following equation:

$$A_{\text{cwp}} = (A_{\text{Si}}^{1/2} - A_{\text{H}_2\text{O}}^{1/2})^2$$

The Hamaker constant of the water and quartz are 5×10^{-20} and $1 \times 10^{-19} \text{ J}$, respectively, so:

$$A_{\text{cwp}} = (A_{\text{Si}}^{1/2} - A_{\text{H}_2\text{O}}^{1/2})^2 = 8.58 \times 10^{-21} \text{ J}$$

Under the condition of 20 °C, the interaction forces between a particle and the capillary wall in the univalent electrolyte solution can be simplified as follows:

$$F_L = 1.43 \times 10^{-2} a_p \times (1 + 0.28z) \times (z + 1.4 \times 10^{-3} z^2)^{-2} \quad (14)$$

$$F_e = -3.34 \times 10^{-3} I^{1/2} \times a_p \times 0.74^{0.5 \times z} \tanh(9.756\psi_1) \tanh(9.756\psi_2) \quad (15)$$

$$F_B = -4.77 \times 10^{-23} a_p \times \sigma_l^6 z^{-8} \quad (16)$$

$$F_h = -6.28 \times a_p \times K_l h \times \exp\left(\frac{-z}{h}\right) \quad (17)$$

4. Entrapment capacity of the filter bed

It can be obtained from the above analysis that the forces between the particle and capillary wall are: F_B , F_e , F_h , F_L and τ , and the total forces perpendicular to the capillary wall are $F_{\text{total}} = \Sigma F = F_L + F_e + F_B + F_h$. Due to the roughness and water film of the particle and capillary surface friction force will be generated when the distance between the particle and capillary wall is in a certain range. The friction force acting between the particle and capillary wall can be calculated as follows:

$$F_f = F_{\text{total}} \times f_c$$

where f_c is the coefficient between the fluid shear force and perpendicular forces, which affect the interaction between the particle and capillary wall. f_c is a function of the particle and capillary surface roughness, their distance, etc., which can be obtained through experiment. The whole interaction forces between the particle and capillary wall are shown in Fig. 3.

Fig. 3. The interaction forces between the particle and capillary wall.

When the saturation stage is achieved (i.e. the maximum adsorption occurred), the total forces acting on a particle adsorbed on the inner surface of the capillary will be zero. And the fluid shear force will equal to the frictional force, while their acting direction is opposite.

$$F_f = \tau \quad (18)$$

The fluid shear force in the capillary can be calculated from Eqs. (4), (6) and (9) (see Appendix A):

$$\tau = 8 \times 10^{-3} \frac{3\alpha(1 - \varepsilon_s) \left[\varepsilon_s d_e - 3 \left(z + \frac{a_p}{2} \right) \alpha(1 - \varepsilon_s) \right] Q}{2\varepsilon_s^3 d_e^2} \quad (19)$$

where ε_s is the saturation porosity of the filter bed, and $z + a_p/2$ means that the shear force is calculated in the position of the particle center.

It can be seen that the equations from Eqs. (14)–(17) have analytical solutions for a certain filtration process, i.e. F_f can be expressed by a definite value. The only unknown term is the porosity in Eq. (19), so the saturation porosity of the filter bed ε_s can be obtained from the computation of Eqs. (14)–(19). The entrapment capacity of the filter bed, i.e. the filtration capacity, can be achieved by multiplying the porosity between the clean and saturation bed with the whole filter bed volume.

5. Results and discussion

5.1. Theoretical analysis

The results computed by equations formulated above are presented in this section. The calculation conditions are described as follows: the filter grains are of spherical form, and their equivalent diameter is 1 millimeter, the surface potential of the particles and the filter grains are all -25 mV, the ionic strength of the solution is $0.01 \text{ mol} \cdot \text{l}^{-1}$, the collision diameter σ_1 is 5 nm , λ_e is 100 nm , and the filtration is operated under the temperature of 20°C , the electrolyte used in the filtration

Fig. 4. Theoretical effect of the particle-capillary wall distance on saturation porosity.

is univalent, the coefficient f_c is 0.2, and the empirical constants K_1 and h are 1×10^{-5} and 0.8, respectively. The theoretical relation curves of the saturation porosity ε_s and the adsorption distance between the particle and capillary surface z , the filtration velocity Q and the saturation porosity ε_s , and their relation with the hydraulic slope J for particles of different diameter are presented in Figs. 4–7. Figs. 4 and 5 are obtained under filtration velocity of 8 m h^{-1} , while Figs. 6 and 7 are computed under conditions that the adsorption distances are 2 and 3 nm for particles of diameter (D_p) 0.5 and $5 \mu m$, respectively. Fig. 8 showed the influence of particle surface potential on the filtration capacity and hydraulic slope, and the computational conditions are the same as mentioned above except for different particle surface potential.

It can be seen from Fig. 4 that the saturation porosity decreases with the increase of the particle diameter, which means better entrapment can be achieved for larger particles. Therefore, certain flocculation and aggregation before the filtration

Fig. 5. Theoretical relation curve of the particle-capillary wall distance and saturation hydraulic slope.

Fig. 6. Theoretical influence of filtration velocity on saturation porosity.

process are necessary to attain high quality outlet water. For the particles of the same diameter, the farther the adsorption distance z (i.e. the distance between the particle and capillary wall when adsorption occurred), the larger the saturation porosity, and the lower the entrapment capacity. As shown in Fig. 6 the saturation porosity changes with the filtration velocity, which results from the increase of fluid shear force in the capillary. Higher shear force leads to lower adsorption according to equilibrium analysis of the forces between the particle and capillary wall.

We can conclude from Fig. 5 that the hydraulic slope decreases with the increase of corresponding porosity for the same filtration flux, which means the higher the porosity, the lower the hydraulic loss. Nevertheless, the hydraulic slope increases with the increase of the filtration velocity despite the fact that the saturation porosity increases at the same time, which can be seen from Fig. 7.

From Fig. 8 we can clearly conclude that the surface potential of particles can influence the filtration capacity greatly. Since the surface potential of the filter grain are supposed to be -25 mV , higher surface potential of the particles will result in higher entrapment capacity, which was

Fig. 7. Theoretical relations between filtration velocity and hydraulic slope.

Fig. 8. Influence of particle surface potential on the filtration.

shown in Fig. 8. For particle surface potential of -15 mV, the saturation porosity is greatly lower than that of -25 mV, while the hydraulic slope is greatly increased at the same time. As high surface potential of the particles makes it easier to collision between the particle and capillary wall, which lead to high filtration speed and capacity.

5.2. Experimental results

The filtration cell was a filter column with inner diameter of 3.2 cm and a length of 100 cm. The packed bed of quartz sand (equivalent diameter is 1 mm and surface shape factor is 1.30) was about 20 cm thick and the original porosity is 0.41.

Certain kaolin (Beijing Chaoyang Xudong Chemistry Company) was add into DI water to produce turbidity about 10 NTU, then the water was coagulated with PAC (Zibo, China). The average diameter of the micro-flocs came into the filter bed is about $5\text{ }\mu\text{m}$ (Multisizer II, Coulter Electronics) with the surface potential about -7 mV (ZetaPlus V3.54, Brookhaven Instrument

Corporation), and since the filter grains will be coated with adsorbed flocs at the end of filtration, the surface potential of the capillary wall was also taken as -7 mV. The ionic strength of the solution was 0.001 mole l^{-1} , which was adjusted by NaNO_3 .

After 50 min filtration, the turbidity of the effluent water almost the same that of the influent water, thus it was considered that the saturation entrapment of filter bed has reached, and then the porosity and the hydraulic slope are measured and calculated.

The parameters used in the theoretical calculations are as follows: the collision diameter σ_1 is 5 nm, λ_e is 100 nm, the operation temperature is $20\text{ }^\circ\text{C}$, the empirical constants K_1 and h are 1×10^{-5} and 0.8, respectively. When the coefficient f_c is 0.175, the comparison of the theoretical and experimental results was presented as Fig. 9.

It can be seen from Fig. 9 that the experimental results are similar to the computational ones, although small deviation can be observed. The saturation porosity of the filter bed was increased

Fig. 9. Comparison of the theoretical and experimental results.

with the increase of the filtration velocity, which meant the decrease of the entrapment capacity. However, the hydraulic slope was increased at the same time despite the higher porosity, which was resulted from higher flow rate in the filter bed.

6. Conclusions

The uniform media packed bed filtration is analyzed in the paper firstly, and the main study is concerned about the saturated filter bed. The fluid state in the pores of porous media is approximated by capillary model and then the shear force distribution is calculated. Secondly, the interaction forces including London–van der Waals attraction force, electric double-layer force, Born repulsion force and structural or hydration force between the particle and capillary wall is introduced into this paper as analytical equations. Thirdly, by equilibrium analysis of the forces acting on a particle adhered on the capillary wall, the analytical equation of the saturation porosity in the filtering bed is obtained and simplified. The maximum entrapment capacity during filtration can be calculated by the equations formulated in this paper. Some theoretical results under different operation conditions are presented, and the relations between operation parameters and the filtration capacity are also discussed. At the end of the paper, the theoretical results are compared with the experimental ones, and the comparison analysis verified the model presented in the paper.

The formulation of the analytical equation in this paper is based on uniform media filtration, and the protecting section of the filter bed is not taken into consideration, which means that the whole packed filter bed attains its saturation adsorption. In actual operation system, a certain part of the filter bed near the bottom is needed to protect the outlet water quality. Therefore, a coefficient f_{emp} should be considered when the formula developed in this paper is adopted to predict the entrapment capacity of the filter bed. The coefficient f_{emp} is a function of the shape factor of filter grain, the length of the protecting section, etc., which should be obtained through in situ experiment.

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Appendix A. The derivation of Eq. (19)

$$d_c = \frac{2\varepsilon}{3\alpha(1-\varepsilon)} d_e \quad (4')$$

$$\bar{u} = \frac{Q}{A_e} \quad (6')$$

It was shown in the paper that A_e is the pore area per unit area of filter medium:

$$A_e = \pi \left(\frac{d_c}{2} \right)^2 n = \varepsilon,$$

(page 5, line 6). When the saturation condition is reached $\varepsilon = \varepsilon_s$, then:

$$d_c = \frac{2\varepsilon_s}{3\alpha(1-\varepsilon_s)} d_e \quad (A)$$

$$\bar{u} = \frac{Q}{\varepsilon_s} \quad (B)$$

$$\tau = 8 \times 10^{-3} \times \bar{u} \times (d_c^{-1} - 2zd_c^{-2}) \quad (9')$$

By substituting Eqs. (A) and (B) into Eqs. 9' and 19' can be obtained:

$$\tau = 8 \times 10^{-3} \frac{3\alpha(1-\varepsilon_s) \left[\varepsilon_s d_e - 3 \left(z + \frac{a_p}{2} \right) \alpha (1-\varepsilon_s) \right] Q}{2\varepsilon_s^3 d_e^2} \quad (19')$$

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