Multifractal pattern and process during a recent period of forest expansion in a temperate mountainous region of China

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ABSTRACT

Understanding the mechanisms that drive spatial pattern change over space and time is a central goal of landscape ecology, but these patterns are often complex. Multifractals, an extension of the concept of fractal geometry, is especially suited to the description of complex pattern. In our paper, we examine the scaling properties of forest spatial pattern over 32 years of forest expansion in a warm temperate deciduous forest near Beijing city using multifractal analysis. An idealized fractal growth model was used to simulate spatial patterns and to link patterns to processes. We show that the despite substantial increases in forest cover, the scaling properties of the spatial pattern of the forest landscape remains similar over time. The variance of Rényi spectra among different years implied that the changes in the forest landscape over time may be a fractal growth process. The fact that these scaling behaviors persist across a heterogeneous landscape suggests that simple rules could exist for some processes of forest expansion, facilitating prediction and further modeling efforts.

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1. Introduction

Understanding landscape spatial pattern change over space and time and finding the underlying governing processes is a central goal of landscape ecology. The ecological importance of this understanding is heightened by the dramatic changes in landscapes that are occurring in response to global ecological changes. Though forest cover has declined in many parts of the world, some regions are exhibiting a “forest transition” in which forest cover has increased over the past few decades, including China (Rudel et al., 2005). There are several processes involved in these observed trends (Nagendra and Southworth, 2010), however very little attempt has been made to quantitatively examining the patterns and processes involved in these dramatic changes in forest cover. In addition, while understanding aggregate indicators such as percent cover are important, other measures may be required to untangle governing spatiotemporal processes.

Ecological patterns and processes and their relationships are dependent on scale (Levin, 1992) and thus analyzing ecological pattern across scales is critical for our understanding of underlying processes (Wiens, 1989). Cross-scale analysis of ecological pattern is a difficult but hot topic in ecology. It has been suggested that finding scaling

laws can be a way to unify natural phenomena (Feigenbaum, 1979). To this end, power law scaling is the most commonly and widely used scaling law in ecology (Arrhenius, 1921; Borda-de-Água et al., 2002; Gaston and Blackburn, 2000; Milne, 1991; West and Brown, 2005; White et al., 2008). The power law is usually related to self-similar (fractal) structures in nature (Harte et al., 1999; Ostling et al., 2004; Zhang et al., 2006). The distribution of species, populations and communities in forest landscapes have been shown to have power-law related fractal properties (Borda-de-Água et al., 2002; Drake and Weishampel, 2000, 2001; Enquist et al., 2009; Harte et al., 1999; Ostling et al., 2004; Pagnutti et al., 2007; Scheuring and Riedi, 1994; West et al., 2009; Zhang et al., 2006) and thus some hope for generalizability of these patterns still exist.

In addition to intrinsic (biotic) factors, the spatial pattern of forests is determined by many extrinsic (often abiotic) factors, such as topography, soil, and climate (Forman, 1995). These factors are themselves fractally distributed in nature (Caniego et al., 2005; Deidda et al., 1999; Kravchenko et al., 2000; Ma et al., 2001; Martinez et al., 2009; Montero, 2005; Zeleke and Si, 2004). The interactions of abiotic and biotic factors increase complexity (Turner and Romme, 1994), thus no single universal scaling factor describing the scaling of the pattern should be expected, but rather a spectrum of scaling exponents (Scheuring and Riedi, 1994) might be observed. Indeed a well-developed technique, namely multifractal analysis, assumes that no single fractal dimension (D) describes patterns observed in nature (Feder, 1988; Milne, 1991; Scheuring and Riedi, 1994). The

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Rényi spectrum (Rényi, 1970) of multifractal theory is a flexible extension of the power law fractal. Milne (1991) suggested that multifractal analysis is a useful tool for describing landscape pattern. It could help in our understanding of different ecological interactions generating patterns, and could be a good indicator of ecological processes in space and time (Carranza et al., 2007; Scheuring and Riedi, 1994).

Recently several studies have examined the processes generate power-laws in vegetation patch patterns using simple cellular automata type models of growth dynamics (Kéfi et al., 2007; Pagnutti et al., 2007; Scanlon et al., 2007). These models have all pointed to the importance of local interactions for generating observed power-laws in landscape patterns. However, while several of these studies attempt to capture single power-laws, some of them present results suggesting that landscapes can vary in fractal dimension according to environmental conditions. Indeed, fractal patterns can also change in time. In general, linking fractal-based ecological indicators to temporal processes remains a challenge in ecology, as many different processes can generate fractal patterns. One approach is to follow indicators over time and examine correlative trends, but this has obvious logistic limits. Another approach is to construct a dynamical model of the process and correlate output with observed patterns. We use the latter approach to examine forest spatial pattern change over time in a mountainous area of Beijing, China. Our aim is determine whether forest spatial patterns, as measured by multifractal indicators, have changed in time in concert with dramatic increases in forest cover and whether these can be linked to simple rules of forest expansion.

2. Methods

2.1. Data sets

2.1.1. Forest spatial data

We use Landsat data to detect the change in spatial patterns of forest cover over time (1978, 1991, 2002 and 2010) of Beijing, China. The municipality is located at 39°56' N and 116°20' E, covers an area of 16,808 km². The four periods of Landsat data are all taken for path 123 and row 32. Except the first period (May 6, 1978) for which MSS data is used with a spatial resolution of 80 m × 80 m, TM data are used for other periods (May 6 1991, May 16, 2002 and May 05,

2010) and the spatial resolution is 30 m × 30 m. All data is interpreted in two classes (forest and non-forest) using supervised classification method in Erdas Image 8.5. The total classification accuracy is 85.96%, assessed by Accuracy Assessment modular of the software.

A 28.8 km × 28.8 km site, nearly 50 km west of Beijing city, is selected for data analysis (Fig. 1). In this area, the climate is a typical warm temperate continental monsoon climate with average annual precipitation of 500–650 mm (from low elevation to high elevation). The mean annual temperature is 5–10 °C. This area also has high heterogeneity in the landforms. Most of the area is more than 600 m above sea level. The vegetation of this region is a secondary deciduous broad-leaved forest (Chen, 1997). Oak (*Quercus liaotungensis* Koidz.), birches (*Betula platyphylla* Suk., *Betula dahurica* Pall., *Betula chinensis* Maxim.), maple (*Acer mono* Maxim.), and poplar (*Populus davidinna* Dode.) are the dominant species in this area.

2.1.2. Fractal growth model: simulated data

The fractal growth model can successfully describe the complicated patterns in nature for they capture the simple physical mechanisms underpinning associated phenomena. Several types of models have been developed to simulate different types of fractal growth phenomena. These models include ballistic aggregation, Eden growth, diffusion-limited aggregation and local growth models (please see Vicsek, 1992 for details). Here we only selected diffusion-limited aggregation (DLA) model to simulate the forest growth process. DLA is one of the most important models of fractal growth, the growth rule is remarkably simple (Witten and Sander, 1981). This model has been used to simulate many different biological processes including the evolution of drainage networks (Masek and Turcotte, 1993), tree root growth (Hauhs et al., 1995), and clonal growth (Sintes et al., 2005). The model, simply explained, involves a random diffusion process combined with some limits which lead to aggregation. We use this model because of its simplicity, but also because the processes of diffusion and aggregation are ones that we consider important for forest spread across a heterogeneous landscape.

A Matlab code DLA simulator is used to generate fractal growth patterns (<http://leto.net/math/dla.php>). We select 60,000 particles on 1920 by 1920 grid as maximum cluster mass to simulate the fractal growth process. Many images (cluster mass 5000, 5100, 5200, 5300,60,000) are generated. Multifractal methods are used to analyze these generated

Fig. 1. The location of the forest sites used in our study.

spatial patterns. In order to compare these with forest distribution data, the DLA generated images are matched with the forest spatial data in the same scale using ArcGIS software.

2.2. Multifractal analysis

In landscape ecology, this following scaling law has been used most commonly to calculate the landscape fractal dimension via the box counting method (Milne, 1991, 1992):

$$N(\delta) \approx \delta^D$$

where $N(\delta)$ is the number of boxes (area or cells) occupied; δ is the size of box; D is the fractal dimension. However, this method represents a special case of spatial pattern analysis, because $N(\delta)$ only examines the presence and absence (0 or 1) of data, not the proportion within a box. This may not be sufficient to describe the complex patterns of landscapes (Milne, 1992; Ricotta, 2000; Scheuring and Riedi, 1994). The multifractal method used here is a derivation of this method (Everstz and Mandelbrot, 1992).

To implement the 'box-counting' method for multifractal analysis of a distribution supported on an interval $I=[a, b]$, a set of different grids with cells of I with equal size is required, and scaling down is the commonly used method (Harte, 2001). First, a set of different grid with rectangular cells of the size δ is used to partition study area. Each cell is characterized by a grid size (δ), and the ratio between forest area of the cell and the forest cover of the site (the relative area of the cell), $\mu_i(\delta)$. Nine cell sizes (δ) are considered in this study, with the total number of cells $N(\delta)$ in the site (Fig. 2). In every site partitioning process, we re-calculate the relative area of both the forest distribution data and the DLA generated data in each cell $u_i(\delta)$ with cell size (δ).

Rényi dimensions are used for data analysis. Rényi dimensions (Hentschel and Procaccia, 1983; Rényi, 1970), D_q , also called generalized dimensions, are defined as:

$$D_q = \lim_{\delta \rightarrow 0} \frac{1}{q-1} \frac{\log \left[\sum_{i=1}^{N(\delta)} \mu_i(\delta)^q \right]}{\log(\delta)} \quad q \neq 1 \quad (1)$$

$$D_1 = \lim_{\delta \rightarrow 0} \frac{\sum_{i=1}^{N(\delta)} \mu_i(\delta) \log(\mu_i(\delta))}{\log(\delta)} \quad q = 1 \quad (2)$$

Parameter q acts as a scanning tool scrutinizing smaller versus larger values of the measure $\mu_i(\delta)$ along the support (Kravchenko et al., 2000). For $q \gg 1$, regions with a high degree of concentration are amplified, while regions with a small degree of concentration are magnified for $q \ll -1$ (Montero, 2005). In order to give a solution of D_q , another formula originating from Eqs. (1) and (2) is introduced:

$$I_q(\delta) = \begin{cases} \frac{1}{q-1} \log \left[\sum_{i=1}^{N(\delta)} \mu_i^q(\delta) \right] & q \neq 1 \\ -\sum_{i=1}^{N(\delta)} \mu_i(\delta) \log[\mu_i(\delta)] & q = 1 \end{cases} \quad (3)$$

where $I_q(\delta)$ called Rényi information.

By changing the size of δ , we can calculate a series $I_q(\delta)$, and plot $\log(\delta)$ against $I_q(\delta)$. The dimensions D_q are estimated by least square fitting of the $\log(\delta)$ – $I_q(\delta)$ plot. The absolute value of the slope of the fitting line is D_q . Coefficients of determination R^2 and significance of those fits are then obtained. This process is repeated for q ranging from -10 to 10 in increments of 0.5 . In plotting q versus D_q , we obtain the Rényi spectra.

Fig. 2. Nine sampling sizes used to partition the site, ranging from (a) 14,400 m by 14,400 m to (i) 600 m by 600 m. For each one, we show the sampling size, δ , the total number of the cells, $N(\delta)$. The size of the site is 28800 m by 28800 m.

3. Results

The forest appears to change over the past few decades in the region surrounding Beijing, China with a clear trend in forest expansion from 1978 to 2010 (Fig. 3(a)–(d)). The forest coverage rate increases gradually, from 17.16% in 1978 to 49.36% in 2010. Forests are naturally distributed at high elevations in this area (Fig. 3(a)–(d) and (e)). This is partly due to low disturbance of human activity and the

favourable environmental conditions (the average annual precipitation is higher at high elevations than at low elevations). Fig. 4 shows the multifractal spectra of the forest spatial pattern in different years; each point of multifractal spectra is statistically significant ($P < 0.0001$). The best and worst fits for value D_q are shown in Fig. 5 (a, b). The D_q values in multifractal spectra increase with time from 1978 to 2010. The left part ($q < 0$) of the spectra is increasing faster than the right part ($q > 0$) of the spectra over time. This means that

Fig. 3. Forest distribution pattern and rate of increase in forest cover in different years. (a) 1978, forest coverage 17.16%; (b) 1991, forest coverage 31.57%; (c) 2002, forest coverage 43.16%; (d) 2010, forest coverage 49.36%. (e) Landform of the forest data site. The elevation unit of legend is meter (m).

Fig. 4. D_q - q curves of forest distribution pattern in different years.

small forest landscape patches are decreasing faster than the increase in large patches. Smaller patches contribute more to the D_q value in the left part of the spectra, and large patches contribute more to the right part of the spectra. The difference ($D_{q_{max}} - D_{q_{min}}$) between maximum value of D_q and minimum values of D_q are 1.3271 (1978), 1.3304 (1991), 1.6278 (2002), and 2.6603 (2010) respectively.

The DLA model generated spatial patterns that gradually expand in space at different fractal growth stages which resemble the landscape forest spatial pattern. We selected four images for which the cluster masses (CM) number is 13,000; 25,500; 34,000; and 60,000 respectively (Fig. 6) as a demonstration because the multifractal spectra are similar to that of the forest distribution pattern spectra ($q < 0$) (Figs. 4; 7). The values of D_q in each spectra show an increasing trend with the fractal growth process, and each point of the multifractal spectra is statistically significant ($P \leq 0.0005$). Fig. 5(c, d) shows the best and worst fits for value D_q of the DLA model generated pattern. The left part of the spectra is also increasing faster than the right part. The difference between the maximum value of D_q and the minimum value of D_q are 1.4365 (CM 13000), 1.5699 (CM 25500), 1.8663 (CM 34000), and 2.2182 (CM 60000) respectively.

4. Discussion

Forest landscapes have complex patterns and processes in space and time (Bradbury et al., 2000; Gisiger, 2001; Klomp and Green, 1996). The spatial-temporal complexity in nature could be described by many methods, but information theory-based power law scaling seems to be an effective tool in describing such complexity (Loehle and Wein, 1994; Ricotta, 2000; Ricotta and Anand, 2006). We find that the scaling laws for forest spatial patterns hold over time (1978, 1991, 2002, and 2010) in our study site, despite dramatic increases in forest cover. This result is interesting in and of itself because it suggests that the expansion of forests in this region is following a process with some constraints. The processes of expansion need to be studied further. Afforestation (planting) is largely the cause of forest cover expansion in China in recent decades (Song and Zhang, 2010). However, most afforestation sites are located in lowland areas and consist of different types of forests (e.g. pine, larch and cypress are the main planting species). Some planting is also done on higher elevations (those of our sites) and could have contributed somewhat to the expansion observed in the area we studied, thus rendering the process both 'natural' and human-induced. At the landscape scale, forest spatial pattern is the result of an integration of climate, landform, disturbance and biotic factors (Turner and Romme, 1994). Further studies could examine longer time series to determine whether there exists transitions or thresholds in this behavior over time in this region and whether the patterns observed also hold in forests undergoing expansion in other parts of the world.

The similarity between the multifractal spectra change over time of forest distribution patterns and the fractal growth model generated patterns implies that forest expansion over time in this region could follow a fractal growth process. The fact that the scaling properties of forest spatial distribution over time is important, as it implies that a simple model could describe the spatial-temporal complexity of landscape evolution (Bolliger et al., 2003). This then suggests that simple rules could be used to model forest dynamics as has been suggested also in other studies (Colasanti et al., 2007; Pagnutti et al., 2007; Sturtevant et al., 2004). Most of the dominant species are dispersed by wind (i.e. birches, maple, poplar) and this may represent some of the 'random' aspect of DLA in this area, but the precise dispersal patterns of the dominant species would need to be further investigated to confirm this. Furthermore, often in forests, growth

Fig. 5. The best and worst fits for values of D_q of forest spatial distribution patterns and DLA simulated patterns. (a) the best fit of forest pattern: $q = 0.5$, year 2002; (b) the worst fit of forest pattern: $q = 10$, year 1978; (c) the best fit of DLA simulated pattern: $q = -10$, cluster mass = 13,000; and (d) the worst fit of DLA simulated pattern: $q = 1.5$, cluster mass = 34,000.

Fig. 6. DLA model generated spatial patterns. (a) Cluster mass = 13,000 particles; (b) Cluster mass = 25,500 particles; (c) Cluster mass = 34,000 particles; and (d) Cluster mass = 60,000 particles.

conditions improve where there are established individuals (Padilla and Pugnaire, 2006), leading to the clustering observed in the growth patterns.

While admittedly the DLA may not be the best model for the relatively recent but ongoing forest expansion process in China (and was used here mainly for illustrative purposes), other models should be developed and tested keeping the above ideas in mind. There are also external factors, not explicitly included in the simple DLA that could be taken into account

to explain the spatial patterns observed: the establishment of seedlings can be spatially limited because habitat conditions are more favourable at high elevations than at low elevations. This may result in the limited aggregation of individuals establishing at higher elevations first and then expanding gradually (Fig. 3(a)–(d) and (e)). Having said this, the work of Tang (2008) could be relevant to examine implications of changing land-use management (e.g., reforestation programs) as it has shown that occupancy probability in the DLA model can strongly affect spatial structure. Kendal (1992) suggested that the tendency of organisms to form aggregations could be thought of as a special case of diffusion-limited aggregation. Furthermore, Maurer and Taper (2002) have argued that that such simple models should be applicable and that “organisms may be aggregated across several spatial scales by a combination of the need to aggregate to reproduce, the ecological limitations inherent in their fundamental niches, and by the nonrandom distribution of ecological conditions imposed by a heterogeneous environment.” Further studies on such problems are suggested, because it could bring us closer to integrating static and dynamic approaches to examining both natural and human-mediated changes in forest systems.

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Fig. 7. D_q - q curves of DLA generated patterns. CM: Cluster Mass.

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